Weak Adhesive High-Level Replacement Categories and Systems: A Unifying Framework for Graph and Petri Net Transformations

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Abstract. Adhesive high-level replacement (HLR) systems have been recently introduced as a new categorical framework for graph tranformation in the double pushout (DPO) approach. They combine the well-known concept of HLR systems with the concept of adhesive categories introduced by Lack and Sobociński.

While graphs, typed graphs, attributed graphs and several other variants of graphs together with corresponding morphisms are adhesive HLR categories, such that the categorical framework of adhesive HLR systems can be applied, this has been claimed also for Petri nets. In this paper we show that this claim is wrong for place/transition nets and algebraic high-level nets, although several results of the theory for adhesive HLR systems are known to be true for the corresponding Petri net transformation systems.

In fact, we are able to define a weaker version of adhesive HLR categories, called weak adhesive HLR categories, which is still sufficient to show all the results known for adhesive HLR systems. This concept includes not only all kinds of graphs mentioned above, but also place/transition nets, algebraic high-level nets and several other kinds of Petri nets. For this reason weak adhesive HLR systems can be seen as a unifying framework for graph and Petri net transformations.

1 Introduction

The use of categorical techniques for unifying frameworks in Computer Science has a long tradition. In the early 1970ies the concept of closed monoidal categories was proposed by Goguen in [1] as a unifying framework for different kinds of deterministic automata. An extension of this framework to nondeterministic and stochastic automata using pseudo-closed categories was presented in [2]. Other important examples are the unifying frameworks of institutions and specification frames respectively. This first framework is based on a categorical treatment of signatures, models and sentences introduced by Goguen and Burstall [3], and the second one in [4] combines directly signatures and sentences to specifications. In both cases we obtain a unifying framework for all kinds of algebraic and logical specification techniques.

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Most recently the unifying framework of adhesive high-level replacement (HLR) systems for different kinds of graph transformation systems has been introduced in [5, 6]. The corresponding concept of adhesive HLR categories integrates those of HLR categories in [7] and adhesive categories by Lack and Sobociński [8], which was later extended to quasi-adhesive categories [9]. The concept of adhesive categories requires the existence of pushouts along monomorphisms and pullbacks, and the property that pushouts along monomorphisms are van Kampen (VK) squares. Roughly spoken the last property means that pushouts are stable under pullbacks and vice versa pullbacks are stable under combined pushouts and pullbacks. In the case of adhesive HLR categories the class of all monomorphisms is replaced by a subclass $\mathcal{M}$ of monomorphisms closed under composition and decomposition and the existence of all pullbacks by pullbacks along $\mathcal{M}$-morphisms. In [5, 6] it is shown that there is a unifying framework of adhesive HLR systems for graph transformation systems based on the double pushout (DPO) approach [10] concerning a large variety of different graph concepts, like labeled graphs, typed graphs, attributed graphs, typed attributed graphs and hypergraphs. The key idea is to show that adhesive HLR categories satisfy a number of different properties, called HLR properties, which are used in [7] to prove important results like the Local Church-Rosser Theorem, the Parallelism Theorem and the Concurrency Theorem. This was first shown for adhesive categories in [8] for the class $\mathcal{M}$ of all monomorphisms and later extended to adhesive HLR categories in [5, 6] and to quasiadhesive categories in [9], where $\mathcal{M}$ is the class of all regular monomorphisms.

The idea to apply the DPO approach to Petri nets was first considered for place/transition nets in [7] and for algebraic high-level nets in [11]. In [5] we have claimed that the category (**PTNets**, $\mathcal{M}$) of place/transition nets with the class $\mathcal{M}$ of all injective morphisms is an adhesive HLR category in order to apply the general theory of adhesive HLR systems also to place/transition nets. Unfortunately this claim is wrong as we show in this paper. The reason is that **PTNets** has general pullbacks, but pullbacks in general cannot be constructed componentwise in **Sets**. However, pullbacks along monomorphisms in **PTNets** can be constructed componentwise in **Sets**. This is the key idea to weaken the concept of adhesive HLR categories using weak VK squares, such that (**PTNets**, $\mathcal{M}$) is a weak adhesive HLR category, and nevertheless this weaker concept still allows to verify the HLR properties used in [7, 5, 6] to prove under some additional assumptions the following main results:
1. Local Church-Rosser Theorem,
2. Parallelism Theorem,
3. Concurrency Theorem,
4. Embedding and Extension Theorem,
5. Local Confluence Theorem - Critical Pair Lemma.

In this paper we show for elementary nets, place/transition nets and algebraic high-level nets that they are weak adhesive HLR categories for a suitable class of morphisms. In [5, 6] we have shown already that adhesive HLR categories satisfy the HLR properties to prove the main results stated above. In this paper we show that this is already true for weak adhesive HLR categories. This implies that the main results are also true for different kinds of Petri net transformation systems including elementary, place/transition and algebraic high-level nets. Note, that in contrast to the "classical" theory of Petri nets and systems based on the token game, where the structure of the nets remains unchanged, the theory of Petri net transformations allows not only the token game, but also to change the structure of the nets. In this sense weak adhesive HLR categories can be seen as a unifying framework not only for graph but also for Petri net transformations.

This paper is organized as follows:
In Section 2 we review adhesive and adhesive HLR categories as introduced in [8] and [5]. In Section 3 we extend these concepts to weak adhesive HLR categories and systems. This is the basis to define Petri net transformation systems as an instance of weak adhesive HLR systems in Section 4.

Acknowledgement

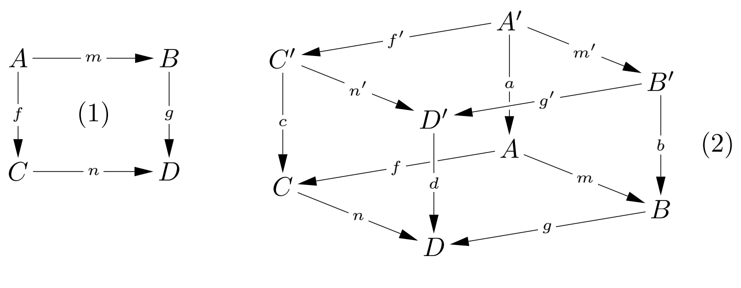
This paper is a contribution to a special issue of Springer LNCS as a Festschrift on the occasion of the 65th birthday of J.A. Goguen. We are glad to announce that the important contributions of J.A. Goguen concerning categorical unifying frameworks for different areas in Computer Science have mainly influenced the school in Berlin leading to similar ones and also to the new unifying framework for graph transformation published recently and for Petri net transformation presented in this paper.

2 Review of Adhesive and Adhesive HLR Categories

The intuitive idea of adhesive categories are categories with suitable pushouts and pullbacks which are compatible with each other. More precisely the definition is based on so-called van Kampen squares.

The idea of a van Kampen (VK) square is that of a pushout which is stable under pullbacks, and vice versa that pullbacks are stable under combined pushouts and pullbacks. The name van Kampen derives from the relationship between these squares and the Van Kampen Theorem in topology (see [12]).

Definition 1 (van Kampen square). *A pushout (1) is a* van Kampen square*, if for any commutative cube (2) with (1) in the bottom and the back faces being pullbacks holds: the top face is a pushout iff the front faces are pullbacks.*

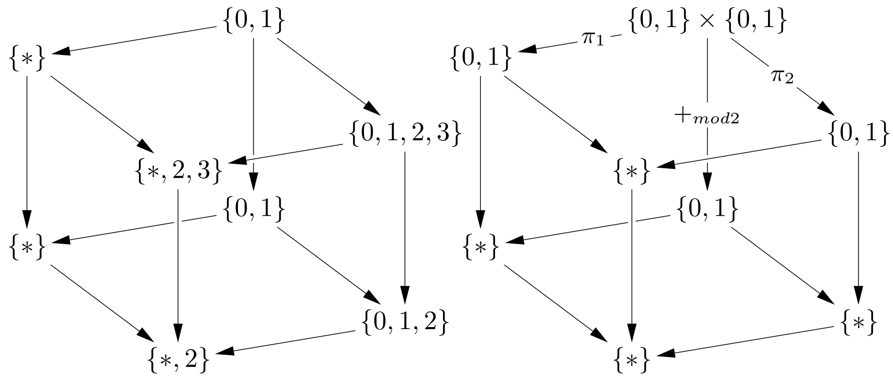


It might be expected that at least in the category **Sets** of sets and functions each pushout is a van Kampen square. Unfortunately this is not true (see Ex. 1). But at least pushouts along monomorphisms (injective functions) are VK squares (see [8, 9]).

Fact 1 (VK squares in Sets). *In* **Sets***, each pushout along a monomorphism is a VK square. Pushout (1) is called a pushout along a monomorphism, if* m *(or symmetrically* f*) is a monomorphism.*

Example 1 (VK squares in **Sets***).* In the following diagram on the left hand side a VK square along an injective function in **Sets** is shown. All morphisms are inclusions, or 0 and 1 are mapped to $*$ and 3 to 2.

Arbitrary pushouts are stable under pullbacks in **Sets**. That means, one direction of the VK square property is also valid for arbitrary morphisms. But the other direction is not necessarily fulfilled. The cube on the right hand side is such a counterexample for arbitrary functions: all faces commute, the bottom and the top are pushouts and the back faces are pullbacks. But obviously the front faces are no pullbacks, therefore the pushout in the bottom fails to be a VK square.



□

In the following definition of adhesive categories only those VK squares of Def. 1 are considered where m is a monomorphism. According to Lack and Sobociński [8] we define

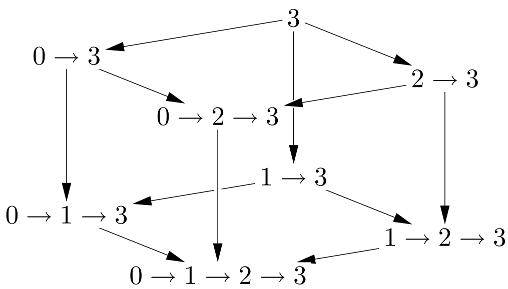
Definition 2 (adhesive category). *A category* **C** *is an* adhesive category*, if*

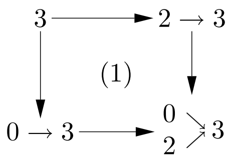
1. **C** *has pushouts along monomorphisms (i.e. pushouts, where at least one of the given morphisms is a monomorphism),*
2. **C** *has pullbacks,*
3. *pushouts along monomorphisms are VK squares.*

Let us first consider some basic examples and counterexamples for adhesive categories (see [8]).

Fact 2 (Sets, Graphs, $\mathbf{Graphs_{TG}}$ as adhesive categories). *The categories* **Sets** *of sets and functions,* **Graphs** *of graphs and graph morphisms and* $\mathbf{Graphs_{TG}}$ *of typed graphs and typed graph morphisms are adhesive categories.*

Counterexample 2 (non-adhesive categories). For example, the category **Posets** of partially ordered sets and the category **Top** of topological spaces and continuous functions are not adhesive categories. In the following diagram a cube in **Posets** is shown that fails to be a van Kampen square. The bottom is a pushout with injective functions (monomorphisms) and all lateral faces are pullbacks, but the top square is no pushout in **Posets**. The proper pushout over the corresponding morphisms is the square (1).





□

Remark 1. In [9] Lack and Sobociński have also introduced a variant of adhesive categories, called quasiadhesive categories, where the class of monomorphisms in Def. 2 is replaced by regular monomorphisms. A monomorphism is called regular, if it is the equalizer of two morphisms. For adhesive and also for quasiadhesive categories Lack and Sobociński have shown, that all the HLR properties, shown for adhesive HLR categories in Thm. 2 below, are valid. This allows to prove several important results of graph transformation systems in the framework of adhesive and also of quasiadhesive categories. On the other hand adhesive and also quasiadhesive categories are special cases of adhesive HLR categories $(\mathbf{C}, \mathcal{M})$ (see Def. 3 below), where the class $\mathcal{M}$ is specialized to the class of all monos and of all regular monos respectively.

The main difference between adhesive HLR categories and adhesive categories is that a distinguished class $\mathcal{M}$ of monomorphisms is considered instead of all monomorphisms, so that only pushouts along $\mathcal{M}$-morphisms have to be VK squares. Moreover, only pullbacks along $\mathcal{M}$-morphisms and not over arbitrary morphisms are required (see [5, 6]).

Definition 3 (adhesive HLR category). *A category* **C** *with a morphism class* $\mathcal{M}$ *is called an* adhesive HLR category*, if*

1. *$\mathcal{M}$ is a class of monomorphisms closed under isomorphisms, composition ($f : A \to B \in \mathcal{M}, g : B \to C \in \mathcal{M} \Rightarrow g \circ f \in \mathcal{M}$) and decomposition ($g \circ f \in \mathcal{M}, g \in \mathcal{M} \Rightarrow f \in \mathcal{M}$),*

2. **C** *has pushouts and pullbacks along* $\mathcal{M}$*-morphisms and* $\mathcal{M}$*-morphisms are closed under pushouts and pullbacks,*
3. *pushouts in* **C** *along* $\mathcal{M}$*-morphisms are VK squares.*

Remark 2. $\mathcal{M}$-morphisms are closed under pushouts if, for a pushout (1) in Def. 1, $m \in \mathcal{M}$ implies that $n \in \mathcal{M}$. Analogously, $\mathcal{M}$-morphisms are closed under pullbacks if, for a pullback (1), $n \in \mathcal{M}$ implies that $m \in \mathcal{M}$.

Example 3 (adhesive HLR categories).

- All adhesive categories are adhesive HLR categories for the class $\mathcal{M}$ of all monomorphisms.
- The category (**HyperGraphs**, $\mathcal{M}$) of hypergraphs with the class $\mathcal{M}$ of injective hypergraph morphisms is an adhesive HLR category.
- Another example for an adhesive HLR category is the category (**Sig**, $\mathcal{M}$) of algebraic signatures with the class $\mathcal{M}$ of all injective signature morphisms.
- The category (**ElemNets**, $\mathcal{M}$) of elementary Petri nets with the class $\mathcal{M}$ of all injective Petri net morphisms is an adhesive HLR category (see Fact 3).
- An important example of an adhesive HLR category is the category ($\mathbf{AGraphs_{ATG}}$, $\mathcal{M}$) of typed attributed graphs with a type graph ATG and the class $\mathcal{M}$ of all injective morphisms with isomorphisms on the data part. □

Counterexample 4 (non-adhesive HLR categories). The categories (**PTNets**, $\mathcal{M}$) of place/transition nets and (**Spec**, $\mathcal{M}$) of algebraic specifications, where $\mathcal{M}$ is the class of all the corresponding monomorphisms, fail to be adhesive HLR categories (see Ex. 6). □

3 Weak Adhesive HLR Categories and Systems

As pointed out in Counterex. 4 the category (**PTNets**, $\mathcal{M}$) of place/transition nets with the class $\mathcal{M}$ of all monomorphisms fails to be an adhesive HLR category. For this reason we introduce now a slightly weaker version, called weak adhesive HLR category.

For a weak adhesive HLR category we only soften item 3 in Def. 3, so that only special cubes are considered for the VK square property.

Definition 4 (weak adhesive HLR category). *A category* **C** *with a morphism class* $\mathcal{M}$ *is called a* weak adhesive HLR category*, if*

1. $\mathcal{M}$ *is a class of monomorphisms closed under isomorphisms, composition and decomposition,*
2. **C** *has pushouts and pullbacks along* $\mathcal{M}$*-morphisms and* $\mathcal{M}$*-morphisms are closed under pushouts and pullbacks,*

3. *pushouts in* **C** *along* $\mathcal{M}$*-morphisms are weak VK squares, i.e. the VK square property holds for all commutative cubes with* $m \in \mathcal{M}$ *and* ($f \in \mathcal{M}$ *or* $b, c, d \in \mathcal{M}$) *(see Def. 1).*

Remark 3. For the weak version of the VK square property it is sufficient to require $f \in \mathcal{M}$ or $b, c, d \in \mathcal{M}$. In both cases this makes sure that the pullback squares in the cube are pullbacks along $\mathcal{M}$-morphisms.

Example 5 (weak adhesive HLR categories).

- All adhesive HLR categories are weak adhesive HLR categories.
- The category (**PTNets**, $\mathcal{M}$) of place/transition nets with the class $\mathcal{M}$ of all monomorphisms is a weak adhesive HLR category (see Fact 4).
- Similarly the category $\mathbf{AHLNets}(\mathbf{SP}, \mathbf{A})$ of algebraic high-level nets with fixed specification SP and algebra A considered with the class $\mathcal{M}$ of injective morphisms is a weak adhesive HLR category (see Fact 5).
- An interesting example of high-level structures, which are not graph-like, are algebraic specifications (see [13]). The category (**Spec**, $\mathcal{M}_{strict}$) of algebraic specifications with the class $\mathcal{M}_{strict}$ of all strict injective specification morphisms is a weak adhesive HLR category. □

Similar to adhesive HLR categories also weak adhesive HLR categories are closed under product, slice, coslice, functor and comma category constructions. That means we can construct new weak adhesive HLR categories from given ones.

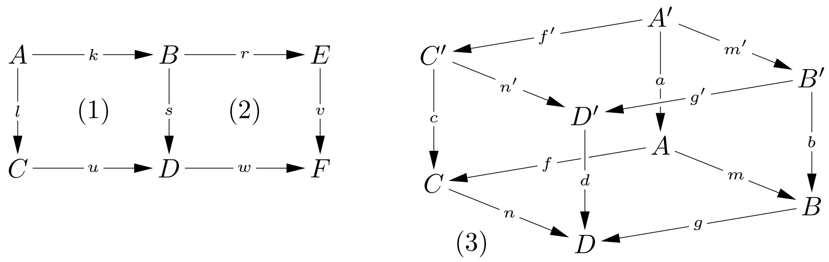
Theorem 1 (construction of weak adhesive HLR categories). *Weak adhesive HLR categories can be constructed as follows:*

1. *If* (**C**, $\mathcal{M}_1$) *and* (**D**, $\mathcal{M}_2$) *are weak adhesive HLR categories, then the product category* ($\mathbf{C} \times \mathbf{D}$, $\mathcal{M}_1 \times \mathcal{M}_2$) *is a weak adhesive HLR category.*
2. *If* (**C**, $\mathcal{M}$) *is a weak adhesive HLR category, so are the slice category* ($\mathbf{C}\backslash X$, $\mathcal{M} \cap \mathbf{C}\backslash X$) *and the coslice category* ($X\backslash\mathbf{C}$, $\mathcal{M} \cap X\backslash\mathbf{C}$) *for any object* X *in* **C**.
3. *If* (**C**, $\mathcal{M}$) *is a weak adhesive HLR category, then for every category* **X** *the functor category* ([**X**, **C**], $\mathcal{M}$*-functor transformations) is a weak adhesive HLR category. An* $\mathcal{M}$*-functor transformation is a natural transformation* $t : F \to G$ *where all morphisms* $t_X : F(X) \to G(X)$ *are in* $\mathcal{M}$.
4. *If* (**A**, $\mathcal{M}_1$) *and* (**B**, $\mathcal{M}_2$) *are weak adhesive HLR categories and* $F : \mathbf{A} \to \mathbf{C}$, $G : \mathbf{B} \to \mathbf{C}$ *are functors, where* F *preserves pushouts along* $\mathcal{M}_1$*-morphisms and* G *preserves pullbacks (along* $\mathcal{M}_2$*-morphisms), then the comma category* ($ComCat(F, G; \mathcal{I})$, $\mathcal{M}$) *with* $\mathcal{M} = (\mathcal{M}_1 \times \mathcal{M}_2) \cap Mor_{ComCat(F,G;\mathcal{I})}$ *is a weak adhesive HLR category .*

In the following theorem we show several important properties for weak adhesive HLR categories, which are essential to prove the main results in Cor. 1. These properties have been required as HLR properties in [7] to show some of the main results for HLR systems. In [8], it was shown already that these HLR properties are valid for adhesive categories. They were extended to adhesive HLR categories in [5], and now also for weak adhesive HLR categories using almost the same proofs.

Theorem 2 (properties of weak adhesive HLR categories). *Given a weak adhesive HLR category* ($\mathbf{C}$, $\mathcal{M}$)*, then the following properties hold:*

1. Pushouts along $\mathcal{M}$-morphisms are pullbacks: *Given the following pushout (1) with $k \in \mathcal{M}$, then (1) is also a pullback.*
2. $\mathcal{M}$ pushout-pullback decomposition lemma: *Given the following commutative diagram with (1)+(2) being a pushout, (2) a pullback, $w \in \mathcal{M}$ and ($l \in \mathcal{M}$ or $u \in \mathcal{M}$). Then (1) and (2) are pushouts and also pullbacks.*
3. Cube pushout-pullback lemma: *Given the following commutative cube (3), where all morphisms in the top and in the bottom are in $\mathcal{M}$, the top is a pullback and the front faces are pushouts. Then we have: the bottom is a pullback iff the back faces of the cube are pushouts.*



4. Uniqueness of pushout complements: *Given $k : A \to B \in \mathcal{M}$ and $s : B \to D$ then there is up to isomorphism at most one C with $l : A \to C$ and $u : C \to D$ such that (1) is a pushout.*

Now we are able to generalize graph transformation systems, grammars and languages in the sense of [10] based on the category **Graphs** to weak adhesive HLR categories, which was already done for HLR, adhesive and adhesive HLR categories in [7], [8] and [5] respectively.

In general, a weak adhesive HLR system is based on productions, also called rules, that describe in an abstract way how objects in this system can be transformed. An application of a production is called direct transformation and describes how an object is actually changed by the production. A sequence of these applications yields a transformation.

Definition 5 (production and transformation). *Given a weak adhesive HLR category* ($\mathbf{C}$, $\mathcal{M}$)*, a* production $p = (L \stackrel{l}{\leftarrow} K \stackrel{r}{\rightarrow} R)$ *(also called rule) consists of three objects L, K and R called left hand side, gluing object and right hand side respectively, and morphisms $l : K \to L$, $r : K \to R$ with $l, r \in \mathcal{M}$.*

Given a production $p = (L \stackrel{l}{\leftarrow} K \stackrel{r}{\rightarrow} R)$ and an object G with a morphism $m : L \to G$, called match. A direct transformation $G \stackrel{p,m}{\Longrightarrow} H$ *from G to an object H is given by the following diagram, where (1) and (2) are pushouts.*

$$\begin{array}{ccccc} L & \xleftarrow{l} & K & \xrightarrow{r} & R \\ \downarrow m & (1) & \downarrow k & (2) & \downarrow n \\ G & \xleftarrow{f} & D & \xrightarrow{g} & H \end{array}$$

A sequence $G_0 \Rightarrow G_1 \Rightarrow ... \Rightarrow G_n$ *of direct transformations is called a* transformation *and is denoted as* $G_0 \stackrel{*}{\Rightarrow} G_n$. *For* $n = 0$, *we have the identical transformation* $G_0 \stackrel{id}{\Rightarrow} G_0$, *i.e.* $f = g = id_{G_0}$. *Moreover, we allow for* $n = 0$ *also isomorphisms* $G_0 \stackrel{\sim}{=} G_0'$, *because pushouts and hence also direct transformations are only unique up to isomorphism.*

Definition 6 (weak adhesive HLR system, grammar and language). *A* weak adhesive HLR system $AHS = (\mathbf{C}, \mathcal{M}, P)$ *consists of a weak adhesive HLR category (*$\mathbf{C}$, $\mathcal{M}$*) and a set of productions* P.

A weak adhesive HLR grammar $AHG = (AHS, S)$ *is a weak adhesive HLR system together with a distinguished start object* S.

The language L *of a weak adhesive HLR grammar is defined by* $L = \{G \mid \exists$ *transformation* $S \stackrel{*}{\Rightarrow} G\}$.

In [5, 6] it is shown that the HLR properties stated in Thm. 2 together with binary coproducts compatible with $\mathcal{M}$ are sufficient to prove the following main results for adhesive HLR systems. Hence we also have the following main results for weak adhesive HLR systems which are stated explicitely in [7] for HLR systems and in [5, 6] for adhesive HLR systems.

Corollary 1 (main results for weak adhesive HLR systems). *Given a weak adhesive HLR system with binary coproducts compatible with* $\mathcal{M}$ *(i.e.* $f, g \in \mathcal{M} \Rightarrow f + g \in \mathcal{M}$*), then we have the following results:*

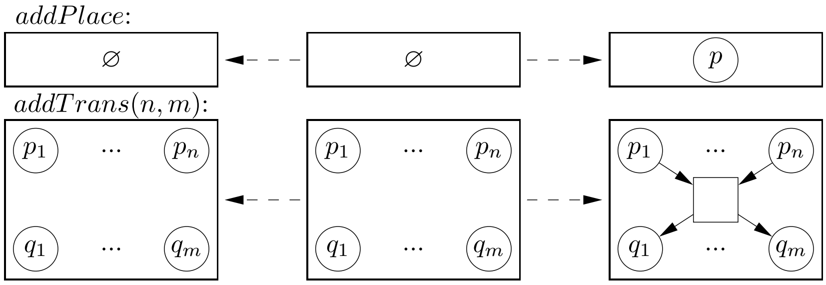
1. *Local Church-Rosser Theorem,*
2. *Parallelism Theorem,*
3. *Concurrency Theorem.*

The Local Church-Rosser Theorem allows one to apply two graph transformations $G \Rightarrow H_1$ via p_1 and $G \Rightarrow H_2$ via p_2 in an arbitrary order leading to the same result H, provided that they are parallel independent. In this case they can also be applied in parallel, leading to a parallel graph transformation $G \Rightarrow H$ via the parallel production $p_1 + p_2$. This second main result is called the Parallelism Theorem. The Concurrency Theorem is concerned with the simultanous execution of causally dependent transformations.

4 Petri Net Transformation Systems

Petri net transformation systems have been first introduced in [7] for the case of low-level nets and in [11] for high-level nets using the algebraic presentation of Petri nets as monoids as introduced in [14]. The main idea of Petri net transformation systems is to extend the well-known theory of Petri nets based on the token game by general techniques which allow to change also the net structure of Petri nets. In [15], a systematic study of Petri net transformation systems has been presented in the categorical framework of abstract Petri nets, which can be instantiated to different kinds of low-level and high-level Petri nets. In this chapter we show that the category (**ElemNets**, $\mathcal{M}$) of elementary Petri nets is an adhesive HLR category (see Fact 3) and that the categories (**PTNets**, $\mathcal{M}$) of place/transition nets and ($\mathbf{AHLNets(SP, A)}$, $\mathcal{M}$) of algebraic high-level nets over (SP, A) are weak adhesive HLR categories (see Fact 4 and 5). The corresponding instantiations of weak adhesive HLR systems lead to different kinds of Petri net transformation systems.

In the following we present a simple grammar $ENGG$ (elementary net graph grammar) for elementary Petri nets, which allows to generate all elementary nets. The start net S of $ENGG$ is empty. We have a production $addPlace$ to create a new place p and productions $addTrans(n,m)$ for $n, m \in \mathbb{N}$ to create a transition with n input and m output places.



The grammar $ENGG$ can be modified to a grammar $PTGG$ (place/transition net graph grammar) for place/transition nets if we replace the productions $addTrans(n,m)$ by productions $addTrans(n,m)(i_1, ..., i_n, o_1, ..., o_m)$, where $i_1, ..., i_n$ resp. $o_1, ..., o_m$ correspond to the arc weights of the input places $p_1, ..., p_n$ resp. the output places $q_1, ..., q_m$.

Definition 7 (elementary Petri net). *An elementary Petri net is given by* $N = (P, T, pre, post : T \to \mathcal{P}(P))$ *with a set* P *of places,* T *of transitions and pre- and post-domain functions* $pre, post : T \to \mathcal{P}(P)$*, where* $\mathcal{P}(P)$ *is the power set of* P*. A morphism* $f : N \to N'$ *in* **ElemNets** *is given by* $f = (f_P : P \to P', f_T : T \to T')$ *compatible with the pre- and post-domain function, i.e.* $pre' \circ f_T = \mathcal{P}(f_P) \circ pre$ *and* $post' \circ f_T = \mathcal{P}(f_P) \circ post$.

Fact 3 (elementary Petri nets as adhesive HLR category). *The category (***ElemNets***,* $\mathcal{M}$*) of elementary Petri nets is an adhesive HLR category, where* $\mathcal{M}$ *is the class of all injective morphisms.*

Proof idea. The category **ElemNets** is isomorphic to the comma category $ComCat(ID_{\mathbf{Sets}}, \mathcal{P}; \mathcal{I})$, where $\mathcal{P}$: **Sets** $\to$ **Sets** is the power set functor and

$\mathcal{I} = \{1, 2\}$. According to Thm. 1.4 it suffices to note that $\mathcal{P} : \mathbf{Sets} \to \mathbf{Sets}$ preserves pullbacks using the fact that ($\mathbf{Sets}$, $\mathcal{M}$) is an adhesive HLR category. □

Definition 8 (place/transition net). *According to [14] a place/transition net* $N = (P, T, pre, post : T \to P^{\oplus})$ *is given by a set* P *of places, a set* T *of transitions, as well as pre- and post-domain functions* $pre, post : T \to P^{\oplus}$*, where* $P^{\oplus}$ *is the free commutative monoid over* P*. A morphism* $f : N \to N'$ *in* **PTNets** *is given by* $f = (f_P : P \to P', f_T : T \to T')$ *compatible with the pre- and post-domain functions, i.e.* $pre' \circ f_T = f_P^{\oplus} \circ pre$ *and* $post' \circ f_T = f_P^{\oplus} \circ post$*.*

Fact 4 (place/transition nets as weak adhesive HLR category). *The category (***PTNets***,* $\mathcal{M}$*) of place/transition nets is a weak adhesive HLR category, but not an adhesive HLR category, if* $\mathcal{M}$ *is the class of all injective morphisms.*

Proof idea. The category **PTNets** is isomorphic to the comma category $ComCat(ID_{\mathbf{Sets}}, \Box^{\oplus}; \mathcal{I})$ with $\mathcal{I} = \{1, 2\}$, where $\Box^{\oplus} : \mathbf{Sets} \to \mathbf{Sets}$ is the free commutative monoid functor. According to Thm. 1.4 it suffices to note $\Box^{\oplus} : \mathbf{Sets} \to \mathbf{Sets}$ preserves pullbacks along injective morphisms using the fact that ($\mathbf{Sets}$, $\mathcal{M}$) is a weak adhesive HLR category. This implies that ($\mathbf{PTNets}$, $\mathcal{M}$) is a weak adhesive HLR category.

It remains to show that ($\mathbf{PTNets}$, $\mathcal{M}$) is not an adhesive HLR category. This is due to the fact, that $\Box^{\oplus} : \mathbf{Sets} \to \mathbf{Sets}$ does not preserve general pullbacks. This would imply that pullbacks in **PTNets** are constructed componentwise for places and transitions. In fact, in Ex. 6 we present a non-injective pullback in **PTNets**, where the transition component is not a pullback in **Sets**, and a cube which violates the VK properties of adhesive HLR categories. □

Example 6 (non-VK square in **PTNets***).* The square (1) in Fig. 1 with non-injective morphisms g_1, g_2, p_1, p_2 is a pullback in the category **PTNets**, where the transition component is not a pullback in **Sets**. In the cube in Fig. 1 the bottom square is a pushout in **PTNets** along an injective morphism $m \in \mathcal{M}$, all side squares are pullbacks, but the top square is no pushout in **PTNets**. Hence we have a counterexample for the VK property. □

In the following we combine algebraic specifications with Petri nets leading to algebraic high-level (AHL) nets (see [11]). For simplicity we fix the corresponding algebraic specification SP and the SP-algebra A. For the more general case, where also morphisms between different specifications and algebras are allowed, we refer to [11]. Under suitable restrictions for the morphisms we also obtain a weak adhesive HLR category in the more general case (see [15] for HLR properties of high-level abstract Petri nets).

Intuitively, an AHL net is a Petri net, where ordinary, uniform tokens are replaced by data elements from the given algebra. Firing a transition t means to remove some data elements from the input places and add some data elements, computed by term evaluation, to the output places of t. There could be also some

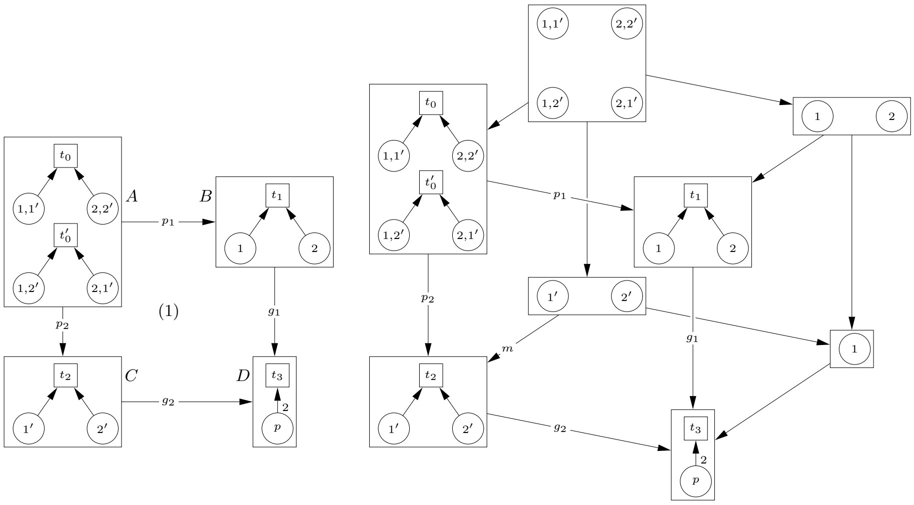


Fig. 1. A pullback and a non-VK square in **PTNets**

firing conditions to restrict the firing behaviour of a transition. In addition, a typing of the places restricts the data elements which could be put on each place to that of a certain type.

Definition 9 (AHL net). *An AHL net over (SP, A), where $SP = (SIG, E, X)$ has additional variables X and $SIG = (S, OP)$, is given by $N = (SP, P, T, pre, post, cond, type, A)$ with sets P and T of places and transitions, $pre, post : T \to (T_{SIG}(X) \otimes P)^{\oplus}$ as pre- and post-domain functions, $cond : T \to \mathcal{P}_{fin}(Eqns(SIG, X))$ assigning to each $t \in T$ a finite set $cond(t)$ of equations over SIG and X, $type : P \to S$ a type function and A an SP-algebra. Note that $T_{SIG}(X)$ is the SIG-term algebra with variables X, $(T_{SIG}(X) \otimes P) = \{(term, p) \mid term \in T_{SIG}(X)_{type(p)}, p \in P\}$ and $\Box^{\oplus}$ is the free commutative monoid functor. A morphism $f : N \to N'$ in $\mathbf{AHLNets(SP, A)}$ is given by a pair of functions $f = (f_P : P \to P', f_T : T \to T')$ which are compatible with the pre, post, cond and type functions as shown below.*

$$\begin{array}{ccccccc}
 & & T & \overset{pre}{\underset{post}{\rightrightarrows}} & (T_{SIG}(X) \otimes P)^{\oplus} & & P \xrightarrow{type} S \\
\mathcal{P}_{fin}(Eqns(SIG, X)) & \overset{cond}{\longleftarrow} T, \; \overset{cond'}{\longleftarrow} T' & \downarrow f_T & & \downarrow (id \otimes f_P)^{\oplus} & & \downarrow f_P \\
 & & T' & \overset{pre'}{\underset{post'}{\rightrightarrows}} & (T_{SIG}(X) \otimes P')^{\oplus} & & P' \xrightarrow{type'} S
\end{array}$$

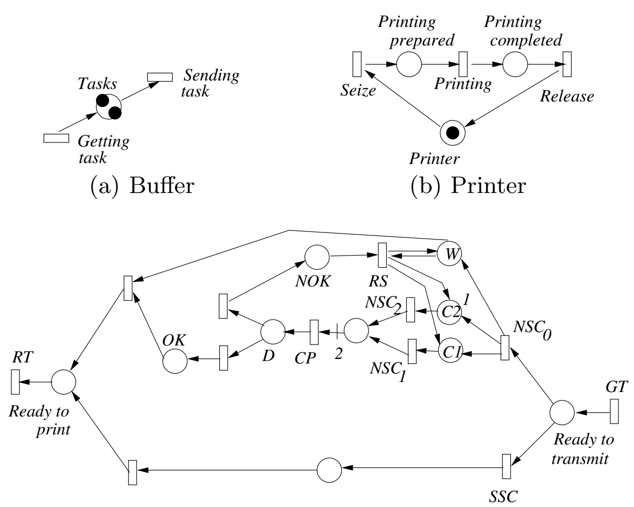
Fact 5 (AHL nets as weak adhesive HLR category). *Given an algebraic specification SP and an SP-algebra A, the category $(\mathbf{AHLNets(SP, A)}, \mathcal{M})$ of algebraic high-level nets over (SP, A) is a weak adhesive HLR category. $\mathcal{M}$ is the class of all injective morphisms f, i.e. f_P and f_T are injective.*

Proof idea. According to the fact that(SP, A) is fixed the construction of pushouts and pullbacks in $\mathbf{AHLNets(SP, A)}$ is essentially the same as in **PTNets**, which is already a weak adhesive HLR category. We can apply the idea of comma categories $ComCat(F, G; \mathcal{I})$, where in our case the source functor of the operations $pre, post, cond, type$ is always the identity $ID_{\mathbf{Sets}}$, and the target functors are $(T_{SIG}(X) \otimes \_)^{\oplus} : \mathbf{Sets} \to \mathbf{Sets}$ and two constant functors. In fact $(T_{SIG}(X) \otimes \_) : \mathbf{Sets} \to \mathbf{Sets}$, the constant functors and $\square^{\oplus} : \mathbf{Sets} \to \mathbf{Sets}$ preserve pullbacks along injective functions. This implies that also $(T_{SIG}(X) \otimes \_)^{\oplus} : \mathbf{Sets} \to \mathbf{Sets}$ preserves pullbacks along injective functions, which is sufficient to verify the properties of a weak adhesive HLR category. $\square$

Corollary 2 (main results for Petri net transformation systems). *The results stated in Cor. 1 are valid for Petri net transformation systems based on the following categories:*

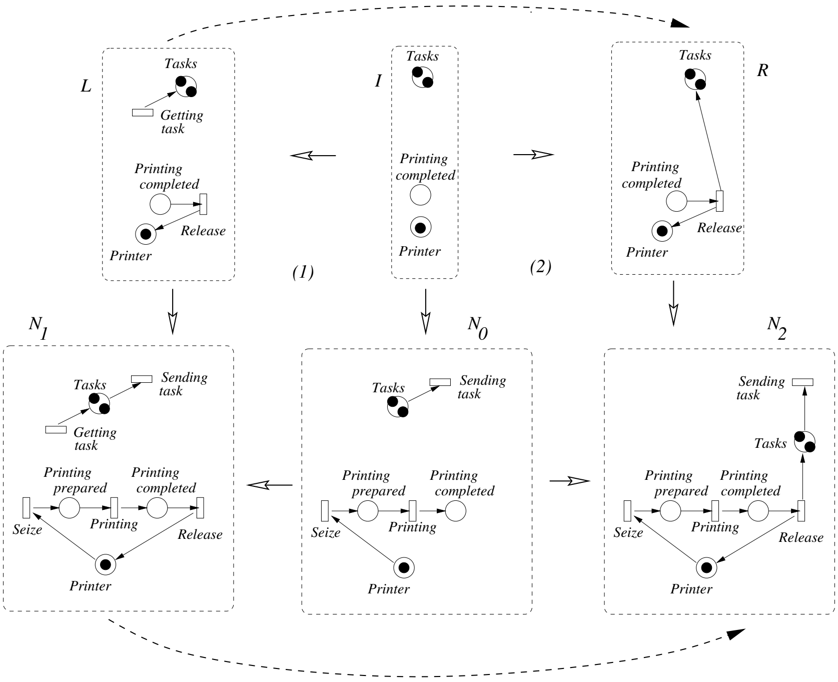
1. *(**PTNets**, $\mathcal{M}$) (see Fact 4),*
2. *(**ElemNets**, $\mathcal{M}$) (see Fact 3),*
3. *(**AHLNets**, $\mathcal{M}$) (see Fact 5).*

Example 7 (place/transition net transformation). We present an example of a place/transition net transformation system from [16], where a communication network is created and analyzed w.r.t. lifeness and safety properties. Here we only consider the construction using Petri net transformations. The system is composed of 3 components: a buffer, a printer and a communication unit depicted in Fig. 2. The behaviour of the buffer and the printer are obvious from the

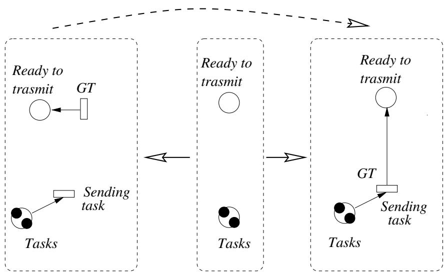


(c) Communication unit

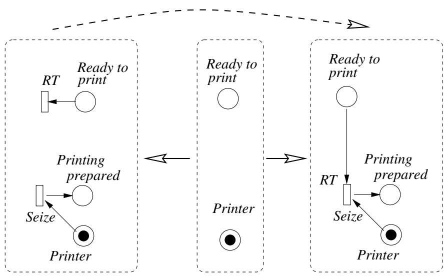
Fig. 2. Components of the system



(a) Buffer – Printer



(b) Buffer – Communication unit



(c) Printer – Communication unit

Fig. 3. Interconnection of components

figure. The communication unit can send a message through a secure (SSC) or non-secure (NSC) channel. Using the NSC channel a message may become corrupted, therefore two copies of the message are sent, which are compared by the receiving subunit D. If both copies differ (NOK), then the transmission has to be repeated, otherwise (OK) it ends.

Petri net transformations are used to connect these three components. In the top row of Fig. 3(a) the production to connect buffer and printer is depicted. Fig. 3(a) shows the whole Petri net transformation as the application of this production to the components buffer and printer. In Fig. 3(b) and Fig. 3(c) the corresponding productions for connecting the communication unit with buffer and printer are shown respectively. Applying all three productions leads to the communication network depicted in Fig. 4.

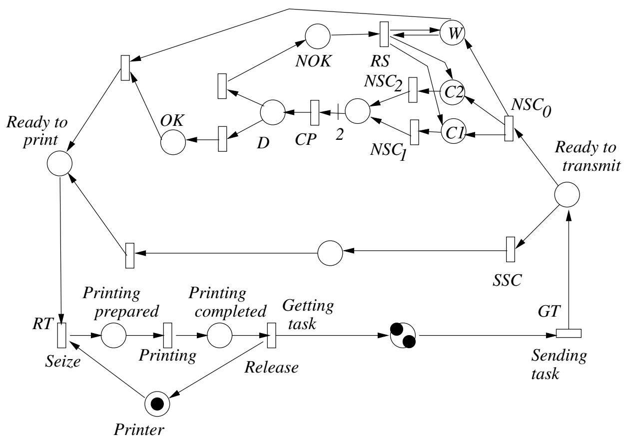


Fig. 4. Resulting communication network

5 Conclusion

In this paper we have shown how to extend adhesive HLR categories and systems - recently introduced as a new categorical framework for graph transformation in [5, 6] - to weak adhesive HLR categories and systems in order to be suitable also as a unifying framework for Petri net transformations. It is interesting to note that all the results for HLR systems based on adhesive HLR categories are still valid under the weaker assumptions of weak adhesive HLR categories. But we might need the stronger assumptions for results based on general pullback constructions as considered in [8, 9]

Especially we have shown in this paper that the category (**PTNets**, $\mathcal{M}$) of place/transition nets with the class $\mathcal{M}$ of all monomorphisms is not an adhesive HLR category, but a weak adhesive HLR category. This is sufficient to show

that the following main results of graph transformation systems are also valid for Petri net transformation systems:
1. Local Church-Rosser Theorem
2. Parallelism Theorem
3. Concurrency Theorem
We conjecture that also the following results
4. Embedding and Extension Theorem
5. Local Confluence Theorem
stated explicitely in [5, 6] for adhesive HLR systems are valid for our Petri net transformation systems considered above. The Embedding and Extension Theorem allows us to embed transformations into larger contexts, and with the Local Confluence Theorem we are able to show local confluence of transformation systems on the basis of the confluence of critical pairs. As additional properties we need a suitable $\mathcal{E}'$-$\mathcal{M}'$ pair factorization and initial pushouts for Petri nets which have been shown for graphs already in [5, 6].

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