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COSPAN DPO APPROACH: AN ALTERNATIVE FOR DPO GRAPH TRANSFORMATIONS

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Abstract

The DPO approach for graph transformations is based on productions $p = (L \leftarrow K \rightarrow R)$ and direct transformations defined by two pushouts, where, roughly spoken, in the first pushout all items in $L \backslash K$ are deleted and in the second one all items $R \backslash K$ are added, while those items in K are preserved. Intuitively, K is the intersection of L and R and, formally, $p = (L \leftarrow K \rightarrow R)$ is a span of graph morphisms.

In this paper, we consider productions $\overline{p} = (L \rightarrow \overline{K} \leftarrow R)$ which are cospans of graph morphisms, and $\overline{K}$ corresponds to the union of L and R. As before, direct transformations are defined by double pushouts, but now the first pushout adds all items in $\overline{K} \backslash L$ and the second one deletes $\overline{K} \backslash R$. This basic idea can be extended to an alternative graph transformation approach, called cospan DPO approach. Key notions of the classical DPO approach

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can be reformulated in the cospan DPO approach and our main result shows in which way corresponding concepts and results are equivalent.

Introduction

The DPO approach for graph transformation has been introduced in [1] and is today one of the most prominent graph transformation approaches concerning theory and applications (see [2, 5, 6] and [7]). It is based on the idea of gluing graphs along designated subgraphs, which can be formalized by the idea of pushouts in the category of graphs. More precisely, a production $p = (L \overset{l}{\leftarrow} K \overset{r}{\rightarrow} R)$ consists of graphs L, K, and R, called left hand side, gluing, and right hand side, resprectively, and two injective graph morphisms l and r. A direct transformation $G \Rightarrow H$ via (p, m) with match $m : L \rightarrow G$ is defined by two pushouts (1) and (2), where in pushout (1) all items $m(L\backslash K)$ in G are deleted leading to graph D, and in pushout (2) all items in $R\backslash K$ are added leading to graph H. Given p and $m : L \rightarrow G$ the context graph D can only be constructed if a suitable gluing condition is satisfied.

$$
\begin{tikzcd}
p : L \arrow[d, "m"'] \arrow[dr, phantom, "(1)"] & K \arrow[l, "l"'] \arrow[r, "r"] \arrow[d] \arrow[dr, phantom, "(2)"] & R \arrow[d] \\
G & D \arrow[l] \arrow[r] & H
\end{tikzcd}
$$

Various modifications of this "classical" DPO concept have been studied in the literature, e.g. the double-pullback approach, where (1) and (2) are pullbacks and not necessarily pushouts [4], the sesqui-pushout approach, where production morphisms may be non-injective and (1) is a certain pullback, but not necessarily a pushout [3], and the DPO approach in adhesive categories [8] or weak adhesive HLR categories [7], where the category of graphs is replaced by suitable other categories, like the category of labeled graphs, typed graphs, hypergraphs, attributed graphs, Petri nets, or algebraic high-level nets.

Moreover, the DPO approach has been implemented in the AGG system for typed attributed graphs including simulation and analysis of graph transformation systems [9].

From the implementation point of view, however, it is sometimes easier to add the new items first and then delete (some of) the old items in the second step. In some sense, this would mean to construct first the pushout (2) and then the pushout (1) in the diagram above. However, this is not possible if production p and match m are given as above. But if we replace p by a cospan $\overline{p} = (L \rightarrow \overline{K} \leftarrow R)$, where $\overline{K}$ corresponds to the union of L and R, we can consider pushouts (3) and

(4) to define a "direct transformation" $G \Rrightarrow H$ via $\overline{p}$ and match m, called cospan direct transformation. This means that we first glue together $\overline{K}$ and G along L in pushout (3), which corresponds to adding $\overline{K} \backslash L$, and in the second step delete $\overline{K} \backslash R$ in pushout (4).

$$\begin{tikzcd} \overline{p} : L \arrow[r, "\bar{l}"] \arrow[d, "m"'] \arrow[dr, phantom, "(3)"] & \overline{K} \arrow[d] \arrow[dr, phantom, "(4)"] & R \arrow[l, "\bar{r}"'] \arrow[d] \\ G \arrow[r] & \overline{D} & H \arrow[l] \end{tikzcd}$$

At first glance, this looks strange because we are not sure whether the pushout complement construction leading to pushout (4) exists. In fact, similar to the classical DPO approach, we now need a "cospan gluing condition" in order to construct pushout (4).

In this paper, we will analyse whether it is possible to develop a cospan DPO approach similar to the classical DPO approach and how far both are equivalent. In Section 1, we introduce some basic concepts and results for the cospan DPO approach for graphs. In Section 2, we extend this approach to other categories and establish the relationship to the classical DPO approach which allows us to give indirect proofs for the results in Section 1. In Section 3, we conclude and give an overview of future work.

1 Basic Concepts of the Cospan DPO Appproach for Graphs

In this section, we introduce some basic concepts and results for the cospan DPO approach for graphs. We do not give direct proofs for these results, but they can be obtained from the equivalence to the classical DPO approach shown in Section 2.

Definition 1.1 (Cospan Production and Direct Transformation)**.** *A* cospan production*, short production,* $\overline{p} = (L \xrightarrow{\bar{l}} \overline{K} \xleftarrow{\bar{r}} R)$ *consists of graphs* L*,* $\overline{K}$*, and* R*, and injective graph morphisms* $\bar{l}$ *and* $\bar{r}$ *which are jointly surjective.*

Given a cospan production $\overline{p}$ *and a match* $m : L \to G$*, a* (cospan) direct transformation $G \Rrightarrow H$ *via* $(\overline{p}, m)$ *consists of the following two pushouts* (1) *and* (2)*, where items are added by pushout* (1) *and deleted by pushout* (2).

$$\begin{tikzcd} \overline{p}: L \arrow[r, "\bar{l}"] \arrow[d, "m"'] \arrow[dr, phantom, "(1)"] & \overline{K} \arrow[d, "\bar{k}"] \arrow[dr, phantom, "(2)"] & R \arrow[l, "\bar{r}"'] \arrow[d, "n"] \\ G \arrow[r] & \overline{D} & H \arrow[l] \end{tikzcd}$$

In order to construct pushout (2), we need the following cospan gluing condition, where the boundary of a morphism $f : A \to A'$ is an inclusion from the smallest subgraph of A such that a pushout complement exists. For a morphism $f : A \to A'$ we want to construct a boundary $b : B \to A$, a boundary object B and a context C leading to a pushout, where roughly spoken, A' is the gluing of C and A along the boundary object.

Definition 1.2 (Cospan Gluing Condition)**.** *Let $B \xrightarrow{b} L$ be the boundary of the match $m : L \to G$, then m satisfies the* cospan gluing condition *w.r.t. $\overline{p} = (L \xrightarrow{\bar{l}} \overline{K} \xleftarrow{\bar{r}} R)$ if there is a morphism $\overline{b}^* : B \to R$ such that $\bar{r} \circ \overline{b}^* = \bar{l} \circ b$.*

$$\begin{tikzcd} B \arrow[r, "b"'] \arrow[rrr, bend left, "\overline{b}^*"] & L \arrow[r, "\bar{l}"'] & \overline{K} & R \arrow[l, "\bar{r}"] \end{tikzcd}$$

Remark. *Note, that $B \xrightarrow{b} L \xrightarrow{\bar{l}} \overline{K}$ is also the boundary of $\bar{k}$ in pushout* (1)*, so that the classical gluing condition is satisfied for $\bar{k}$ w.r.t. $\bar{r}$, which allows to construct the context graph H in pushout* (2)*.*

Analogously to the classical case, cospan direct transformations can be constructed iff the cospan gluing condition is satisfied, and are unique up to isomorphism.

Fact 1.3 (Construction and Uniqueness of Direct Transformations)**.** *Given a cospan production $\overline{p}$ and a match $m : L \to G$ then there is a direct transformation $G \Rrightarrow H$ via $(\overline{p}, m)$ iff the cospan gluing condition is satisfied for m w.r.t. $\overline{p}$. Moreover, in this case $\overline{D}$ and H in pushouts* (1) *and* (2) *above are uniquely determined up to isomorphism.*

Once we know how to construct direct transformations in the cospan DPO approach we define as usual transformations as sequences of direct transformations. Now we can study under which conditions we are able to obtain the basic results of the classical DPO approach, like the Local Church-Rosser, Parallelism, Concurrency and Embedding Theorems, also in the cospan DPO approach. In this section, we only consider the Local Church-Rosser Theorem which is based on parallel and sequential independence.

Definition 1.4 (Cospan Parallel Independence)**.** *Two direct transformations $G \Rrightarrow H_1$ via $(\overline{p_1}, m_1)$ and $G \Rrightarrow H_2$ via $(\overline{p_2}, m_2)$ are called* (cospan) parallel independent *if there are morphisms $m_1' : L_1 \to H_2$ and $m_2' : L_2 \to H_1$ such that $y_2 \circ m_1' = x_2 \circ m_1$ and $y_1 \circ m_2' = x_1 \circ m_2$.*

$$\begin{tikzcd}
R_1 \arrow[r] \arrow[d] & \overline{K_1} \arrow[d] & L_1 \arrow[l] \arrow[dr, "m_1"'] \arrow[rrrrd, "m_1'"] & & L_2 \arrow[r] \arrow[dl, "m_2"] \arrow[llllld, "m_2'"'] & \overline{K_2} \arrow[d] & R_2 \arrow[l] \arrow[d] \\
H_1 \arrow[r, "y_1"'] & \overline{D_1} & & G \arrow[ll, "x_1"'] \arrow[rr, "x_2"'] & & \overline{D_2} & H_2 \arrow[l, "y_2"']
\end{tikzcd}$$

Remark. *Sequential independence is defined in a similar way.*

The existence of m_1' and m_2' means that we obtain a match from L_1 of $\overline{p_1}$ to H_2 and a match from L_2 of $\overline{p_2}$ to H_1. These matches will allow us to construct direct transformations $H_2 \Rrightarrow H$ via $(\overline{p_1}, m_1')$ and $H_1 \Rrightarrow H$ via $(\overline{p_2}, m_2')$.

Theorem 1.5 (Local Church-Rosser Theorem)**.** *Given parallel independent direct transformations $G \Rrightarrow H_1$ via $(\overline{p_1}, m_1)$ and $G \Rrightarrow H_2$ via $(\overline{p_2}, m_2)$ there is a graph H and direct transformations $H_2 \Rrightarrow H$ via $(\overline{p_1}, m_1')$ and $H_1 \Rrightarrow H$ via $(\overline{p_2}, m_2')$ such that the sequences become sequentially independent, and vice versa.*

$$\begin{tikzcd}
 & H_1 \arrow[dr, Rrightarrow, "(\overline{p_2}, m_2')"] & \\
G \arrow[ur, Rrightarrow, "(\overline{p_1}, m_1)"] \arrow[dr, Rrightarrow, "(\overline{p_2}, m_2)"'] & & H \\
 & H_2 \arrow[ur, Rrightarrow, "(\overline{p_1}, m_1')"'] &
\end{tikzcd}$$

2 Equivalence of Classical and Cospan DPO Approach

The basic concepts and results of Section 1 can be reformulated for adhesive high-level replacement (HLR) systems based on (weak) adhesive HLR categories $(\mathbf{C}, \mathcal{M})$ (see [7]). For this purpose, we replace graphs by the objects of the category $\mathbf{C}$, injective graph morphisms by morphisms of a morphism class $\mathcal{M}$ of monomorphisms, and jointly surjective graph morphisms by jointly epimorphic morphisms. Thus, a cospan production $p = (L \xrightarrow{\bar{l}} \overline{K} \xleftarrow{\bar{r}} R)$ consists of objects L, $\overline{K}$, and R in $\mathbf{C}$, where $\bar{l}$ and $\bar{r}$ are $\mathcal{M}$-morphisms and jointly epimorphic.

It is possible to weaken the condition of jointly epimorphic morphisms of a cospan production. A cospan of $\mathcal{M}$-morphisms is called a generalized production.

Definition 2.1 (Generalized Cospan Production). *A generalized cospan production $\overline{\overline{p}} = (L \xrightarrow{\overline{\overline{l}}} \overline{\overline{K}} \xleftarrow{\overline{\overline{r}}} R)$ consists of objects L, $\overline{\overline{K}}$, and R, and $\mathcal{M}$-morphisms $\overline{\overline{l}}$ and $\overline{\overline{r}}$.*

Since we do not require jointly epimorphic morphisms for the cospan gluing condition or the construction of pushouts, a cospan direct transformation over a generalized production exists and is defined analogously to the normal cospan production. Moreover, for every generalized cospan production $\overline{\overline{p}}$ there is a cospan production $\overline{p}$ such that $G \Rrightarrow H$ via $(\overline{\overline{p}}, m)$ if and only if $G \Rrightarrow H$ via $(\overline{p}, m)$. This implies that it is sufficient to consider normal cospan productions instead of generalized ones.

Definition 2.2 (Closure of Generalized Cospan Production). *Given a generalized cospan production $\overline{\overline{p}} = (L \xrightarrow{\overline{\overline{l}}} \overline{\overline{K}} \xleftarrow{\overline{\overline{r}}} R)$ then the* closure *of $\overline{\overline{p}}$ is a cospan production $\overline{p} = (L \xrightarrow{\overline{l}} \overline{K} \xleftarrow{\overline{r}} R)$ such that* (1) *is the pullback of $\overline{\overline{l}}$ and $\overline{\overline{r}}$, and* (2) *is the pushout of l and r.*

$$
\begin{tikzcd}
 & K \arrow[dl, "l"'] \arrow[dr, "r"] & \\
L \arrow[dr, "\overline{\overline{l}}"'] \arrow[rr, phantom, "(1)"] & & R \arrow[dl, "\overline{\overline{r}}"] \\
 & \overline{\overline{K}} &
\end{tikzcd}
\qquad
\begin{tikzcd}
 & K \arrow[dl, "l"'] \arrow[dr, "r"] & \\
L \arrow[dr, "\overline{l}"'] \arrow[rr, phantom, "(2)"] & & R \arrow[dl, "\overline{r}"] \\
 & \overline{K} &
\end{tikzcd}
$$

Theorem 2.3 (Equivalence of $\overline{p}$ and $\overline{\overline{p}}$). *In a weak adhesive HLR category with effective pushouts, given a generalized cospan production $\overline{\overline{p}} = (L \xrightarrow{\overline{\overline{l}}} \overline{\overline{K}} \xleftarrow{\overline{\overline{r}}} R)$, its closure $\overline{p} = (L \xrightarrow{\overline{l}} \overline{K} \xleftarrow{\overline{r}} R)$, and a match $m : L \to G$, then we have that*

$$G \Rrightarrow H \text{ via } (\overline{p}, m) \iff G \Rrightarrow H \text{ via } (\overline{\overline{p}}, m).$$

Remark. *Effective pushouts means that for a pullback* (1) *and a pushout* (2) *as above, with all morphisms in $\mathcal{M}$, the induced morphism $\overline{K} \to \overline{\overline{K}}$ is also an $\mathcal{M}$-morphism.*

For a cospan production $\overline{p}$ with jointly epimorphic morphisms, the closure of $\overline{p}$ is the cospan production $\overline{p}$ itself (up to isomorphism), since a pullback over jointly epimorphic $\mathcal{M}$-morphisms is already a pushout in weak adhesive HLR categories with effective pushouts.

Proof. Due to (2) being a pushout, (1) being a pullback and effective pushouts there is an induced morphism $\overline{K} \to \overline{\overline{K}}$ which is an $\mathcal{M}$-morphism.

"⇒" Given $G \Rrightarrow H$ via $(\overline{p}, m)$ then we have the back faces as pushouts in the following cube. We construct the pushout $\overline{\overline{D}}$ over $\overline{D} \leftarrow \overline{K} \rightarrow \overline{\overline{K}}$ and obtain the cospan direct transformation $G \Rrightarrow H$ via $(\overline{\overline{p}}, m)$ by pushout composition.

$$
\begin{tikzcd}
 & & \overline{K} \arrow[dl, shift left=0] \arrow[ddl] \arrow[ddd] & \\
L \arrow[urr, "\bar{l}"] \arrow[dr, "\bar{\bar{l}}"'] \arrow[ddd] & & & R \arrow[ul, "\bar{r}"'] \arrow[dll, "\bar{\bar{r}}"] \arrow[ddd] \\
 & \overline{\overline{K}} \arrow[ddd] & & \\
 & & \overline{D} \arrow[ddl] & \\
G \arrow[urr] \arrow[dr] & & & H \arrow[ul] \arrow[dll] \\
 & \overline{\overline{D}} & &
\end{tikzcd}
$$

"⇐" Given $G \Rrightarrow H$ via $(\overline{\overline{p}}, m)$ we have the front faces as pushouts in the above cube. We construct $\overline{D}$ as pushout of $G \leftarrow L \rightarrow \overline{K}$, get an induced morphism $\overline{D} \rightarrow \overline{\overline{D}}$ and by pushout decomposition $\overline{\overline{D}}$ is the pushout of $\overline{D} \leftarrow \overline{K} \rightarrow \overline{\overline{K}}$. Similarly, we can construct a pushout $\overline{D}'$ in the right hand side of the cube, and by uniqueness of pushout complements with $\overline{K} \rightarrow \overline{\overline{K}} \in \mathcal{M}$ we have that $\overline{D}$ and $\overline{D}'$ are isomorphic leading to the cospan direct transformation $G \Rrightarrow H$ via $(\overline{p}, m)$. □

Starting with a classical production $p = (L \xleftarrow{l} K \xrightarrow{r} R)$ we obtain an adjoint cospan production $\overline{p} = (L \xrightarrow{\bar{l}} \overline{K} \xleftarrow{\bar{r}} R)$ by pushout construction in (1), and vice versa p is obtained by $\overline{p}$ by pullback construction as in the second step of the construction of the closure.

Definition 2.4 (Adjoint Productions). *A classical production* $p = (L \xleftarrow{l} K \xrightarrow{r} R)$ *and a cospan production* $\overline{p} = (L \xrightarrow{\bar{l}} \overline{K} \xleftarrow{\bar{r}} R)$ *are* adjoint *to each other if diagram* (1) *is both pushout and pullback.*

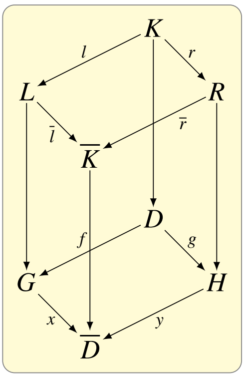
$$
\begin{tikzcd}
 & K \arrow[dl, "l"'] \arrow[dr, "r"] & \\
L \arrow[dr, "\bar{l}"'] \arrow[rr, phantom, "(1)"] & & R \arrow[dl, "\bar{r}"] \\
 & \overline{K} &
\end{tikzcd}
$$

The most interesting question now is how cospan transformations and classical transformations are related to each other. In the following, we show that direct transformations and hence also transformation sequences correspond to each other uniquely (up to isomorphism) for graphs and weak adhesive HLR categories.

Theorem 2.5 (Equivalence of Cospan and Classical DPO Transformation)**.** *Given adjoint productions $p = (L \overset{l}{\leftarrow} K \overset{r}{\rightarrow} R)$ and $\overline{p} = (L \overset{\bar{l}}{\rightarrow} \overline{K} \overset{\bar{r}}{\leftarrow} R)$, and a match $m : L \to G$ then we have the following equivalence of direct transformations in the cospan and in the classical DPO approach:*

$$G \Rrightarrow H \text{ via } (\overline{p}, m) \iff G \Rightarrow H \text{ via } (p, m).$$

Proof. "$\Rightarrow$" Given $G \Rrightarrow H$ via $(\overline{p}, m)$, we have the front faces and the top face as pushouts and pullbacks in the following cube. Now we construct D as pullback in the bottom face, and all morphisms in the top and bottom faces are $\mathcal{M}$-morphisms. The bottom pullback implies a morphism $K \to D$ such that the cube commutes, and the cube pushout-pullback lemma (see [7]) implies that also the back faces are pushouts. These pushouts lead to the direct transformation $G \Rightarrow H$ via (p, m).



"$\Leftarrow$" Given $G \Rightarrow H$ via (p, m), we have the back faces and the top face of the above cube as pushouts and pullbacks. Now we construct $\overline{D}$ as pushout in the bottom face, and get an induced morphism $\overline{K} \to \overline{D}$ such that the cube commutes. By pushout composition and decomposition also the front faces are pushouts. These pushouts lead to the cospan direct transformation $G \Rrightarrow H$ via $(\overline{p}, m)$. □

This result leads to the equivalence of the cospan and the classical DPO approach.

Theorem 2.6 (Equivalence of Cospan and Classical DPO Approach)**.** *For each concept and result in the classical DPO approach there is an adjoint concept and result in the cospan DPO approach, and vice versa.*

Proof Idea. The equivalence is based on the adjointness of productions (see Def. 2.4) and the equivalence of direct transformations defined by the pushout-pullback cube in the proof of Thm. 2.5. □

In the following, we illustrate Thm. 2.6 for the concepts of gluing condition and parallel independence and the corresponding results in both approaches.

Fact 2.7 (Equivalence of Gluing Conditions)**.** *Given adjoint productions* $p = (L \xleftarrow{l} K \xrightarrow{r} R)$ *and* $\overline{p} = (L \xrightarrow{\overline{l}} \overline{K} \xleftarrow{\overline{r}} R)$*, and a match* $m : L \to G$ *then* m *satisfies the cospan gluing condition w.r.t.* $\overline{p}$ *(see Def. 1.2) if and only if* m *satisfies the classical gluing condition w.r.t.* p*.*

Proof. For the boundary B of m, we have to show the equivalence of the classical gluing condition (C) $\exists b^* : B \to K : l \circ b^* = b$ and the cospan gluing condition $(\overline{C})$ $\exists \overline{b}^* : B \to R : \overline{r} \circ \overline{b}^* = \overline{l} \circ b$.

$$
\begin{tikzcd}
 & & K \arrow[dl, "l"'] \arrow[dr, "r"] & \\
B \arrow[r, "b"'] \arrow[urr, bend left, "b^*"] \arrow[rrr, bend left=70, "\overline{b}^*"] & L \arrow[dr, "\overline{l}"'] \arrow[rr, phantom, "(1)"] & & R \arrow[dl, "\overline{r}"] \\
 & & \overline{K} &
\end{tikzcd}
$$

"$(C) \Rightarrow (\overline{C})$": Define $\overline{b}^* := r \circ b^*$, then we have that $\overline{r} \circ \overline{b}^* = \overline{r} \circ r \circ b^* = \overline{l} \circ l \circ b^* = \overline{l} \circ b$.

"$(\overline{C}) \Rightarrow (C)$": $(\overline{C})$ and (1) being a pullback implies that there is a unique b^* with $r \circ b^* = \overline{b}^*$ and $l \circ b^* = b$.

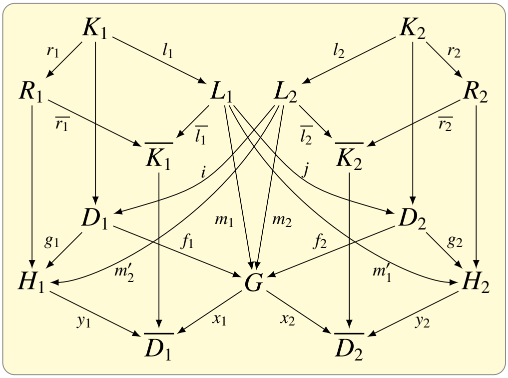
□

Note that Fact 1.3 in Section 1 is the adjoint result corresponding to the well-known construction and uniqueness of direct transformations in the classical DPO approach (see [7]). The proof of Fact 1.3 follows directly from Thm. 2.5 and Fact 2.7, but could also be given directly in the cospan DPO approach.

Fact 2.8 (Equivalence of Parallel Independence)**.** *Two cospan direct transformations* $G \Rrightarrow H_i$ *via* $(\overline{p_i}, m_i)$ *for* $i = 1, 2$ *are parallel independent (see Def. 1.4) if and only if the adjoint direct transformations* $G \Rightarrow H_i$ *via* (p, m_i) *are parallel independent in the classical DPO approach.*

Proof. Given the cospan direct transformations $G \Rrightarrow H_i$ and the adjoint direct transformations $G \Rightarrow H_i$ for $i = 1, 2$ as shown below,we have to show that the conditions for parallel independence in the classical case (C) $\exists i : L_2 \to D_1$:

$f_1 \circ i = m_2 \wedge \exists j : L_1 \to D_2 : f_2 \circ j = m_1$ and in the cospan approach $(\overline{C})$ $\exists m_1' : L_1 \to H_2 : y_2 \circ m_1' = x_2 \circ m_1 \wedge \exists m_2' : L_2 \to H_1 : y_1 \circ m_2' = x_1 \circ m_2$ are equivalent.



"$(C) \Rightarrow (\overline{C})$": Define $m_2' := g_1 \circ i$, then we have that $y_1 \circ m_2' = y_1 \circ g_1 \circ i = x_1 \circ f_1 \circ i = x_1 \circ m_2$, and analogously $m_1' := g_2 \circ j$ leading to $(\overline{C})$.

"$(\overline{C}) \Rightarrow (C)$": The left bottom face being a pullback and $(\overline{C})$ implies that there exists a unique i such that $g_1 \circ i = m_2'$ and $f_1 \circ i = m_2$, and analogously there is j leading to (C). □

Thm. 1.5 in Section 1 is the adjoint result corresponding to the well-known local Church-Rosser Theorem in the classical DPO approach (see [7]). The proof of Thm. 1.5 follows directly from Thm. 2.6, Fact 2.8, and the similar equivalence of sequential independence, but could also be given directly in the cospan DPO approach.

3 Conclusion

In this paper, we have shown that similar to the classical DPO approach based on productions as spans $p = (L \leftarrow K \rightarrow R)$ there is an alternative cospan DPO approach based on productions as cospans $\overline{p} = (L \rightarrow \overline{K} \leftarrow R)$. We have presented some basic concepts and results in this alternative approach in Section 1 and have shown in Section 2 in which sense both approaches are equivalent.

It remains open to analyse the benefits of the cospan DPO approach in more detail from the theoretical and the practical point of view. Especially it is interesting to note that Thm. 2.6 corresponds in some sense to the duality principle in category theory, where spans are replaced by cospans but not pushouts by pullbacks.

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